Model Updating for Validation: A Tutorial

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Model Validation
Establishing Credibility [Thacker]

- **Verification**
  - Credibility from understanding the mathematics
  - Are the equations being solved correctly?
  - Compare computed results to known solutions

- **Validation**
  - Credibility from understanding the physics
  - Are the correct equations being solved?
  - Compare computed results to experimental data

- **Uncertainty Analysis**
  - Credibility from understanding the uncertainties
  - How accurate is the model prediction?
  - Quantify uncertainty & variability from all sources

⇒ Better decision-making capability

Mathematical Evidence

Experimental Evidence

Statistical Evidence
Validation vs Updating

- **Model validation** is the process of deciding the degree of credibility of model predictions for a given application.

- **Model updating** is a subtask of model validation whose objective is to reduce the level of epistemic uncertainty affecting model parameters through the use of relevant experimental data.
Model Updating

Global versus Local Methods

Global methods

- Phenomenological in nature
- Physical interpretation of parametric correction not generally possible
- Predictive capability low in modified configurations

Local methods

- Based on knowledge model which respects structural connectivity
- Model parametrization with respect to model design variables
- Parametric modifications with physically meaningful constraints
- A glimmer of hope in predicting behavior under untested configurations

⇒ Only local methods presented today
**Problem**

- Model updating is a very costly task
- How do I decide if an updated model will significantly improve the prediction of some global system performance?
Model Updating
Impact on System Performance
Model Updating
Impact on System Performance

\[ f_5 \]

\[ \frac{r_c}{f_{\text{nom}}} \]

Incertitude
Model Updating

Flow Chart

Numerical model

Verification of numerical model

Optimal test preparation

Adaptation analysis - tests

Comparison analysis - test

Sensitivity analysis

Model error localisation

Model parameter selection

Parametric estimation

Convergence test

Model reanalysis

Stop

Dynamic testing

Extraction of experimental model
Model Verification

Typical criteria

- Total mass and inertia checks
- Credible static and modal shapes
- Mesh convergence
- Coherence with prototype
- Impact of reduced component models on global behavior...
Model Verification
Impact of Reduced Component Models
Model Verification

Impact of Reduced Component Models

Equation of motion of a component

\[
\begin{bmatrix}
K_{jj} & K_{ji} \\
K_{ij} & K_{ii}
\end{bmatrix}
- \lambda
\begin{bmatrix}
M_{jj} & M_{ji} \\
M_{ij} & M_{ii}
\end{bmatrix}
\begin{bmatrix}
y_j \\
y_i
\end{bmatrix}
= \begin{bmatrix}
f_j \\
0
\end{bmatrix}
\]

Craig-Bampton transformation

\[
T = \begin{bmatrix}
I_{jj} & 0 \\
\Psi_{ij} & \Phi_{ik}
\end{bmatrix}
\]

System réduction

\[
T^T(K - \lambda M)Tq = \begin{bmatrix}
f_j \\
0
\end{bmatrix}
\]

Craig-Bampton superelement

\[
\begin{bmatrix}
\bar{K}_{jj} & 0 \\
0 & \Lambda_k
\end{bmatrix}
- \lambda
\begin{bmatrix}
\bar{M}_{jj} & L_{jk} \\
L_{kj} & I_k
\end{bmatrix}
\begin{bmatrix}
y_j \\
q_k
\end{bmatrix}
= \begin{bmatrix}
f_j \\
0
\end{bmatrix}
\]
Model Verification

Impact of Reduced Component Models

**Inject approximate solution into exact system**

\[(K - \tilde{\lambda}_k M)\tilde{y}_k = \Delta f_k\]

**Evaluate residual displacements**

\[r_k = K^{-1} \Delta f_k\]

**Error indicator of \(k^{th}\) superelement**

\[e_k^k = \left(r^k_k\right)^T K_k r^k_k\]
Model Verification

Impact of Reduced Component Models

Slave modes: 0 - 600 Hz
⇒ Twice band of interest

Frequency errors < 40%
Test Planning
What to Test and How?

Problem

- Make test design decisions based on non-validated model behavior
- Optimally place sensors/actuators to satisfy performance criteria: observability, distinguishability, excitability, and subdomain visibility of targeted structural behaviors
- Define optimal boundary and mass loading conditions in order to sensitise the influential zones for a given application
- Establish testing and validation priorities for subassemblies and components
Test Planning
HRG Example

Base excitation tests

Modal tests
Base excitation tests

Simple dynamics governed by a few global modes with high effective mass

Modal tests

Complex dynamics influenced by numerous local modes
Test Planning

Decision-making indicators

**Observability** → Can we detect the presence of a target response?

**Distinguishability** → Can we distinguish between different target responses?

**Excitability** → Do the input loads inject enough energy into target responses?

**Visibility** → Can a subdomain be localized if it is in error?
Test Planning

Observability

\[ \kappa^i_{\nu} = \frac{\| r Y^i_{\nu} \|_\infty}{\| T Y^i_{\nu} \|_\infty} = \frac{\text{Maximum observed displacement}}{\text{Maximum calculated displacement}} \]

\[ \kappa^i_{\nu} \approx 0 \rightarrow \text{mode is not observable} \]
\[ \kappa^i_{\nu} \approx 1 \rightarrow \text{mode is well observable} \]
Optimal pickup positions [Lallement]

\[ \text{Solve: } \min_{L_i^q} g = \text{cond}(L_i^q Y^{(m)}) \]

\[ L_i^q \in \mathbb{R}^{q,N} \quad \text{Localizes the } i^{\text{th}} \text{ combination of } q \text{ pickups} \]

\[ \Rightarrow \text{Search for sub-optimal solution} \]
Test Planning
Distinguishability

MAC Matrix
All linear DOFs

MAC Matrix
26 optimal pickups
Test Planning

Excitability

Optimal exciter positions [Link]

**Hypothesis:**

\[
L^q \in \mathbb{R}^{q,N}
\]

localizes the q optimal pickups

\[
F = L^q M Y^{(m)}
\]

modal excitation forces limited to q pickups

**Solve:**

\[
\min_{L^e_i} g = \text{cond}(L^e_i F)
\]

with

\[
L^e_i \in \mathbb{R}^{e,q}
\]
Excitability wrt number of exciters

Multivariate Mode Indicator Function (MMIF)
Test Planning
Subdomain Visibility

**Problem**

- *A priori* verification criteria for a model error localization procedure based on Constitutive Equation Error
- Provides **necessary** (but not sufficient) condition for the modelling error in a zone to be detectable
- Pre-processing tool to define an optimal sensor design which depends uniquely on the model connectivity
- Post-processing tool to evaluate the confidence in localization results obtained with a given sensor distribution
- Decision tool to evaluate the gain in parameter visibility with additional sensors and test configurations
Test Planning
Subdomain Visibility

Example

Localization criteria

\[ e_{i\nu} = r_v^T K_i r_v \]

\[ V_i = \max_q \left( \frac{q^T S^T K_i S q}{q^T S^T K S q} \right) \]

\[ S = K^{-1} F \]

Smearing of the error on neighboring well modeled zones
Test Planning

Deterministic vs Robust

Auto MAC

Optimal Deterministic Design

Impact of Model Uncertainty

Auto MAC

Test Planning
Test Planning
Deterministic vs Robust

Optimal versus robust designs

<table>
<thead>
<tr>
<th></th>
<th>Nominal</th>
<th>$\alpha = 0.1$</th>
<th>$\alpha = 0.2$</th>
<th>$\alpha = 0.3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best</td>
<td>1.74</td>
<td>1.48</td>
<td>1.47</td>
<td>1.46</td>
</tr>
<tr>
<td>Worst</td>
<td>1.74</td>
<td>21.3</td>
<td>23.1</td>
<td>27.9</td>
</tr>
<tr>
<td>Robust</td>
<td>1.74</td>
<td>2.14</td>
<td>3.98</td>
<td>10.0</td>
</tr>
</tbody>
</table>

- Optimal solutions have zero robustness – the only way to go is down!
- Robust designs are sub-optimal but will satisfy a desired level of performance for a given level of uncertainty
Test-Analysis Adaptation
Common Reference Frames

**Node Reduction**

1. Create of a global common reference frame
2. Pair homologous nodes based on a nearest neighbor criteria
3. Create local geometric transformations, node by node, to project analytical responses along measurement directions
Test-Analysis Adaptation

Front Bearing Housing (FBH) Example

Common Reference Frame

Node Pairing

Local Node Frames
Test-Analysis Adaptation

Node Reduction

Refined versus crude mesh
Test-Analysis Comparison

Problem

- Quantify the distance between test-analysis behaviors
- Detect and correct for cabling and calibration errors
- Establish a matching between homologous eigenvectors
- Filter non-physical numerical and experimental eigensolutions
- Existence of identified eigensolutions which are not predicted by model
- Existence of analytical eigensolutions which are not identified from measurements
Test-Analysis Comparison

Quantitative Criteria

Relative Error:
\[ C^\lambda_{\nu} = \frac{\lambda^{(m)} - \lambda^{(r)}}{\lambda^{(m)}} \]

MAC:
\[ C^{MAC}_{\nu\sigma} = \frac{(y_{\nu}^{(m)T} W y_{\sigma}^{(r)})^2}{\| y_{\nu}^{(m)} \|^2_W \| y_{\sigma}^{(r)} \|^2_W} \]

MSF:
\[ C^{MSF}_{\nu\sigma} = \frac{\| y_{\nu}^{(m)T} y_{\sigma}^{(r)} \|^2_W}{\| y_{\nu}^{(m)} \|^2_W} \]
Test-Analysis Comparison

FBH Example

Baseline Correlation

Corrected* Correlation

* Correction required due to inversion of several sensor cables!
Test-Analysis Comparison

Holographic Data

Numerical model

Experimental model

Displacements along camera axis

500
400
200
0
-200
-400
Test-Analysis Comparison
Holographic Data

Numerical Model

Experimental Model

MAC

Holographic Data

Test-Analysis Comparison
Holographic Data

Numerical Model

Experimental Model

MAC

Holographic Data
Test-Analysis Comparison

Multiple Eigenvalues

Subspace rotation

\[ Y^A = \begin{bmatrix} Y_1^A & Y_2^A & \ldots & Y_q^A \end{bmatrix} \]

Decomposition into subspaces based on a frequency criteria

Projection of experimental modes on subspaces

With:

\[ y^E \quad H \quad Y^A \quad Y_i^A \in \mathbb{R}^{n,1} \quad \mathbb{R}^{n,N} \quad \mathbb{R}^{N,m} \quad \mathbb{R}^{N,m_i} \]

Identified eigenvector

Test-analysis reduction matrix

Analytical model basis

Decomposed analytical basis
Test-Analysis Comparison

Multiple Eigenvalues

Subspace rotation
Problem

- Understand how strain and kinetic energies are distributed in the structure
- Study the influence of design variables on a targeted set of model features
- Reinitialize the insensitive design variables
Sensitivity Analysis
Local Methods

Subdomain energies

\[ s_{iuv}^K = \frac{1}{\lambda(m)} y_{uv}^{(m)}^T K y_{uv}^{(m)} \]

and

\[ s_{iuv}^M = y_{uv}^{(m)}^T M_i y_{uv}^{(m)} \]

Normalized Element Modal Energies
Model = FEM, Variable = NPshell113_1ref, Sort = bigstram
Sensitivity Analysis

Global Methods

Methods

- Sample design space using Experimental Design, Monte-Carlo, Latin Hypercube, ...
- Analyze correlation between inputs and outputs
- Principal Component Analysis to estimate effective order of design space

Advantages

- Allows very large parametric variations wrt local approach
- Output features is not limited to energies ⇒ complex composite quantities can be analyzed

Disadvantage

- High calculation cost
Sensitivity Analysis

Reinitialization of Model Parameters

Problem

- Quantification of local interface stiffnesses is often difficult to evaluate due to significant geometric simplifications.
- Local optimization procedures require objective functions to be reasonably sensitive to design variables.
- Manual reinitialization based on modal behaviors often inconclusive → insensitive parameter due to displacement field or nominal parameter value?
- Automatic procedure based on sensitizing inputs.
Sensitivity Analysis
Reinitialization of Model Parameters

**Sensitizing inputs:**

\[ Kx = f \]
\[ f_1 = e_i \quad \text{and} \quad f_2 = e_j \]

**Reduction basis:**

\[ B = [K^{-1}f_1 \ K^{-1}f_2] \in \mathbb{R}^{N,2} \]

**Reduced model:**

\[ (K_G + \Delta K_G)q = B^T f \]
\[ K_G = B^T K B \in \mathbb{R}^{2,2} \]
\[ \Delta K_G = B^T \Delta K B \in \mathbb{R}^{2,2} \]

**Relative energy:**

\[ \mathcal{M}(\Delta K) = \frac{q^T \Delta K_G q}{q^T (K_G + \Delta K_G) q} \]
Sensitivity Analysis
Reinitialization of Model Parameters
Localization of Modelling Errors

Problem

- Provide diagnostic tool to encourage a critical analysis of translation of the conceptual model into a numerical model
- Detect dominant modelling errors which are responsible for test-analysis distances
- Regularization of the parametric identification problem by reduction of the number of unknowns
Localization of Modelling Errors

Qualitative Methods

*A priori Localization*

- Study different phases of modelling and the problems encountered (lack of CAD, geometric simplifications, homogenization of mechanical and material properties, missing information, …)

- Detailed examination of prototype to detect gross differences wrt CAD and model (geometry, connectivity, equipment status, …)
**Localization of Modelling Errors**

**Quantitative Method**

**Constitutive Equation Error** [Ladevèze et al]

**Error function:**

\[
\min g = r_v^T K r_v + \| \Delta y_v \|_W
\]

**Solve:**

\[
\begin{bmatrix}
- (K + \omega^2 M) & K - \omega^2 M \\
0,5 (K - \omega^2 M) & \alpha LT WL
\end{bmatrix}
\begin{bmatrix}
r_v \\
y_v^E
\end{bmatrix} = \begin{bmatrix} 0 \\
\alpha LT_c W \dot{y}_v^E
\end{bmatrix}
\]

**Element group error:**

\[
\epsilon_i = \frac{r_v^T (K_i + \omega^2 M_i) r_v}{y_v^T (K_i + \omega^2 M_i) y_v}
\]

where:

\[
K_i = \bigcup_{j \in D_i} K_{j}^{ele}
\]

\[
M_i = \bigcup_{j \in D_i} M_{j}^{ele}
\]
Visibility of Modeling Errors

FBH Example

<table>
<thead>
<tr>
<th>zone</th>
<th>Visibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.78</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
</tr>
<tr>
<td>3</td>
<td>0.57</td>
</tr>
<tr>
<td>4</td>
<td>0.14</td>
</tr>
<tr>
<td>5</td>
<td>0.74</td>
</tr>
<tr>
<td>6</td>
<td>0.84</td>
</tr>
<tr>
<td>7</td>
<td>0.97</td>
</tr>
<tr>
<td>8</td>
<td>0.43</td>
</tr>
</tbody>
</table>
Visibility & Localization Indicators
Problem

- An updated model will be physically meaningful only if the model parametrization is *coherent* with the true modelling errors.
- A model is said to be *equivalent* if the corrected modelling errors are *incoherent* → physics poorly represented.

Incoherent modelling

- Stiffness and mass eccentricities neglected
- Idealized boundary conditions
- Simplification of 3D non-uniform stress state as 2D mono-directional
- Non-converged mesh
**Problem**

- Formulate an objective function based on test-analysis distances as a function of design variables
- Introduce normalizations, weightings, regularization schemes,...
- Estimate values of design variables minimizing the objective function
Objective function: \[ g = \sum \omega_{uv} \left| \lambda_u^{(r)} - \lambda_u^{(m)} \right| + \omega_{uv} \left\| y_u^{(r)} - Ly_u^{(m)} \right\|_W \]

Solve:
\[
\begin{bmatrix}
S_{\lambda_1,p_1} & \cdots & S_{\lambda_1,p_{np}} \\
\vdots & \ddots & \vdots \\
S_{\lambda_m,p_1} & \cdots & S_{\lambda_m,p_{np}} \\
L_{S_{y_1,p_1}} & \cdots & L_{S_{y_1,p_{np}}} \\
\vdots & \ddots & \vdots \\
L_{S_{y_{nm},p_1}} & \cdots & L_{S_{y_{nm},p_{np}}}
\end{bmatrix}
\begin{bmatrix}
\Delta p_1 \\
\vdots \\
\Delta p_{np}
\end{bmatrix}
= 
\begin{bmatrix}
\lambda_1^{(r)} - \lambda_1^{(m)} \\
\vdots \\
\lambda_{nm}^{(r)} - \lambda_{nm}^{(m)} \\
y_1^{(r)} - Ly_1^{(m)} \\
\vdots \\
y_{nm}^{(r)} - Ly_{nm}^{(m)}
\end{bmatrix}
\]

\[ \Rightarrow \quad S \Delta p = \Delta b \quad \text{with} \quad G \Delta p \leq 0 \]
**Parametric Estimation**

**Constitutive Equation Error**

**Solve:**

\[
\min_{p \in P} g = \sum_{\nu} r_{\nu}(p)^T K(p)r_{\nu}(p) + \Delta y_{\nu}^T W \Delta y_{\nu}
\]

- Only introduce parameters corresponding to dominant modelling errors at a given iteration
- Very costly non-linear constrained optimization problem
- Use model reduction techniques for better efficiency
Parametric Estimation

Multi-configuration Updating

Modal Static

C4 parameters

Modal Static

X9 parameters

Interface parameters

Sx=e
Problem

- Model updating is a phase of the model validation process → estimate model design variables to reduce model-test distances
- Numerical modelling → tradeoffs between precision and cost
- Coherent vs incoherent modelling
- Is it possible to correctly identify certain model parameters in the presence of uncertainty with respect to others?
- Can we design a decision-making tool to investigate the robustness of different identification algorithms?

Applications

- Quality control of monitoring systems under unknown structural degradation
- Decision-making in model validation (modelling, test preparations, algorithms)
Evaluated updating algorithms

- Residual based on the Constitutive Equation Error

\[
\min_{p \in \mathcal{P}} g = \sum_{\nu} r_{\nu}(p)^T K(p) r_{\nu}(p) + \Delta y_{\nu}^T W \Delta y_{\nu}
\]

\[
r_{\nu}(p) = K(p)^{-1} \left[ K(p) - \tilde{\omega}_{\nu}^2 M(p) \right] \tilde{y}_{\nu}
\]

- Residual based on Output Error

\[
\min_{p \in \mathcal{P}} g = \sum_{\nu} \Delta f_{\nu}(p) W_f \Delta f_{\nu}(p) + \Delta y_{\nu}^T W_y \Delta y_{\nu}
\]
Parametric Estimation

Robustness to Unmodelled Physics

Constitutive Equation Error Algorithm

![Graph showing the relationship between robustness and design parameter error]
Parametric Estimation

Robustness to Unmodelled Physics

Constitutive Equation Error Algorithm

Output Error algorithm!
Model Reanalysis

Problem

- Evaluate features of numerical model at a point in the design space
- Reduce analysis cost using approximate methods

Methods

- Exact analysis
- Taylor series expansions
- Meta-models (response surfaces, neural networks,...)
- Model reduction \( y_v = [ Y \ K^{-1} \Delta K_1 Y \ K^{-1} \Delta K_2 Y \ ... ] c_v \)
Model Reanalysis

Model reduction

ITERATIVE PROCEDURE

<table>
<thead>
<tr>
<th>Parameterisation of numerical model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modified model ( K(p) ; M(p) \in \mathbb{R}^{N,N} )</td>
</tr>
<tr>
<td>Physical model ( Z(p,\lambda_0) y_0 = 0 )</td>
</tr>
<tr>
<td>Reduced model ( T[Z(p,\lambda_0)] T\epsilon_0 = 0 )</td>
</tr>
<tr>
<td>Analysis</td>
</tr>
<tr>
<td>New parametric hypothesis</td>
</tr>
</tbody>
</table>

PREPARATION

<table>
<thead>
<tr>
<th>Nominal Model ( K(p^0) ; M(p^0) \in \mathbb{R}^{N,N} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A \text{ priori parameterisation of numerical model} )</td>
</tr>
<tr>
<td>Transformation ( y_0 = T\epsilon_0 \text{ avec } T \in \mathbb{R}^{N,n} )</td>
</tr>
</tbody>
</table>
## Model Reanalysis

### Example

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameters</th>
<th>Initial Stiffness</th>
<th>Modified Stiffness</th>
</tr>
</thead>
<tbody>
<tr>
<td>DESVAR_1</td>
<td>CELAS2 510082, CELAS2 510083</td>
<td>7E+5</td>
<td>700</td>
</tr>
<tr>
<td>DESVAR_2</td>
<td>CELAS2 510092, CELAS2 510093</td>
<td>7E+5</td>
<td>4.9E+4</td>
</tr>
<tr>
<td>DESVAR_3</td>
<td>CELAS2 510151</td>
<td>1E+9</td>
<td>1E+5</td>
</tr>
<tr>
<td>DESVAR_4</td>
<td>CELAS2 510152, CELAS2 510153</td>
<td>3E+5</td>
<td>6000</td>
</tr>
</tbody>
</table>
# Model Reanalysis Example

<table>
<thead>
<tr>
<th>Nominal Exact</th>
<th>Perturbed Exact</th>
<th>Approximate Analysis Modal Basis</th>
<th>Approximate Analysis Enriched Modal Basis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (Hz)</td>
<td>Frequency (Hz)</td>
<td>Variation</td>
<td>Frequency (Hz)</td>
</tr>
<tr>
<td>43.99</td>
<td>45.47</td>
<td>3.4 %</td>
<td>43.96</td>
</tr>
<tr>
<td>44.70</td>
<td>46.74</td>
<td>4.5 %</td>
<td>44.66</td>
</tr>
<tr>
<td>53.39</td>
<td>59.34</td>
<td>11.1 %</td>
<td>53.28</td>
</tr>
<tr>
<td>79.77</td>
<td>72.74</td>
<td>-8.8 %</td>
<td>--</td>
</tr>
<tr>
<td>107.96</td>
<td>95.28</td>
<td>-11.7 %</td>
<td>105.27</td>
</tr>
<tr>
<td>123.13</td>
<td>122.96</td>
<td>-0.1 %</td>
<td>123.09</td>
</tr>
</tbody>
</table>
Model Reanalysis

Example

CPU Time

<table>
<thead>
<tr>
<th>Analysis Type</th>
<th>CPU Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact Analysis</td>
<td>48 s</td>
</tr>
<tr>
<td>Preparation of Static Bases</td>
<td>155 s</td>
</tr>
<tr>
<td>Approximate Reanalysis</td>
<td>17 s</td>
</tr>
</tbody>
</table>

Minimum of 5 analyses required before overall speed up
Accessing Model Quality

- Reduction in output errors
- Reduction in Constitutive Equation Errors
- Increased number of matched modes
- Improved correlation for unused responses
- Simultaneous reduction of model distances under multiple configurations
- Physically meaningful parameter values
Limitations of Model Updating

- Distinguishability of geometric and material properties in same subdomain can be difficult to obtain
- Very large number of unknown parameters
- Very difficult to bound prediction errors in untested configurations (geometry, connectivity, BC’s, parameter modifications,...)
- How good is good enough?
Conclusions

- Need to create a rational environment for managing the whole model validation process of which **model updating** is only a single phase
- Decision-making tools provide basis for handling a wider range of candidate options with greater visibility given to the important compromises
- Importance of working with clearly defined objectives for model **usability** - how good is good enough for a given application?
- Importance of validating models under near operating conditions...